A stochastic convective approach to account for model uncertainty due to unresolved humidity variability

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[1] Forecast uncertainty can result from the neglect of humidity variability on spatial scales not resolved by forecast models. To account for this, a stochastic convective scheme for the European Centre for Medium-Range Weather Forecasts (ECMWF) ensemble prediction system is presented that uses the subgrid humidity distributions provided by the cloud scheme. Each member of a forecast ensemble randomly samples this distribution to perturb the convective parcel’s initial humidity and/or the humidity of the air entrained during ascent. Accounting for humidity variability with the new scheme has a smaller impact on tropical ensemble spread in the short range compared to the ECMWF operational scheme that represents uncertainty because of parameterization error. Combining the two schemes to account for both parameterization error and subgrid humidity variability simultaneously generally improves the skill of the operational system for most variables in the short to medium range in midlatitudes, while results in the tropics are mixed, with a notable deterioration in medium-range probabilistic skill for temperature and zonal wind. This deterioration is a consequence of the methodology employed. Since the convective scheme is highly nonlinear, providing zero-mean humidity perturbations to the scheme’s input profiles does not lead to zero-mean perturbations to the output tendencies. For some parameters, this results in an increased bias of the ensemble mean and a deterioration in probabilistic skill. As future convective schemes are optimized to maximize deterministic forecast skill, the methodology will require modification to ensure zero-mean convective output perturbations.


\section{Introduction}

[2] Forecast uncertainty is often categorized in terms of uncertainty in the initial conditions and model error, such as parameterization, parameter and truncation errors [Palmer, 2000; Orrell, 2005]. Model errors are addressable by adding stochastic perturbations to the model physics parameterization schemes’ output tendencies appropriate for the assessed magnitude of uncertainty [e.g., Buizza et al., 1999].

[3] One parameterization of a physical process which often receives attention with regard to stochastic physics schemes is convection. Small but finite changes to boundary layer or free-tropospheric thermodynamic properties which alter the convective inhibition (CIN) can determine whether or not deep convection occurs. Deep convection could potentially be an efficient vehicle for the rapid amplification of small-scale perturbations if the net convective effect can influence synoptic-scale circulations.

[4] In the tropics, convective activity can trigger African easterly waves [Mekonnen et al., 2006], for example, and the way in which convection amplifies convectively coupled Kelvin waves [Emanuel, 1994; Straub and Kiladis, 2003] and the Madden Julian Oscillation [Kiladis et al., 2005] is a subject of active research. Using a toy model of the tropics, Mapes [2000] demonstrated rapidly growing waves if convective activity was predominantly determined by a CIN-based trigger.

[5] In the midlatitudes, Hohenegger and Schär [2007] examined the predictability of convection in an observed case study over Switzerland using a cloud resolving model and found that small (and non-local) temperature perturbations could lead to large changes in organized convection in low-predictability cases. The study of Zhang et al. [2003] examined the predictability of the January 2000 rapidly intensifying storm that affected the North-East coast of the United States. They showed that the failure to predict this system stemmed from the influence of the embedded deep convection. Buizza and Chessa [2002] confirmed that the use of stochastic physics to provide parameterization (including convection) tendency perturbations enabled some of the ensemble prediction system (EPS) members of the European Centre for Medium-Range Weather Forecasts...
(ECMWF) forecast model to provide improved predictions of this storm.

It appears justifiable, therefore, to consider convection an important contributor to forecast uncertainty, which warrants parameterization in a stochastic framework. Uncertainty in convective heating can arise from deficiencies in the parameterization itself. In this case, the level of uncertainty is likely to be larger when the scheme itself is producing large tendencies. Thus the scheme introduced by 

Buizza et al. [1999] multiplies convective parameterization thermodynamic and dynamic output tendencies by a random number. Likewise, a stochastic convective scheme introduced by Lin and Neelin [2000, 2002, 2003] randomly scales the convective closure parameter of convective available potential energy (CAPE) and thus also estimates uncertainty to be proportional to convective activity. Shutts and Palmer [2007] confirmed the validity of this approach by sampling large-domain cloud resolving models output and finding heating rate fluctuations approximately proportional to net heating rate.

Shutts and Palmer [2007] also point out that uncertainty is non-zero even when net convective heating rates are zero, which is not accounted for by the Buizza et al. [1999] scheme. This leads to a second cause of uncertainty in convective activity, referred to by Shapiro and Thorpe [2004], arising from the lack of knowledge of fluctuations of temperature and humidity on scales smaller than the mesoscale model grid, which were demonstrated as key in the study of Zhang et al. [2003]. For example, a grid column with a conditionally unstable thermodynamic profile might have a slightly cooler boundary layer than the critical threshold to initiate deep convection. The convection parameterization thus produces zero tendencies and schemes such as Buizza et al. [1999] will consequentially also assess a zero uncertainty, despite the fact that a small positive perturbation to the boundary layer temperature maybe enough to overcome the convection inhibition, releasing the atmospheric potential energy through the instigation of deep convection. Accounting for subgrid-scale fluctuations of temperature and moisture therefore results in finite uncertainty even if the unperturbed model predicts zero convective activity.

The aim of this work is to introduce a simple framework into the ECMWF ensemble prediction system to account for convective uncertainty because of the subgrid-scale thermodynamic variability, extending the preliminary investigation of Tompkins [2005]. The subgrid-scale thermodynamic variability will be derived consistently with the forecast models’ prognostic variables, which is used to perturb the convective scheme’s input parameters. After introducing both the existing and new schemes in the following section, section 3 examines the impact of the scheme in terms of ensemble spread and probabilistic forecast skill scores. It will be seen that supplementing the existing scheme of Buizza et al. [1999], which accounts for parameterization error, with the new scheme that assess convective uncertainty because of subgrid humidity fluctuations, can improve ensemble skill, but that in the tropics probabilistic skill for some parameters deteriorates in the medium range. This is demonstrated to be the result of the methodology of perturbing input to a highly nonlinear scheme such as the convection scheme. In the conclusions (section 4) it is suggested that corrective measures will be required to ensure zero-mean tendency perturbations for this approach to be ultimately successful.

2. Stochastic Convection Schemes and Experimental Descriptions

2.1. Base Convection Scheme

The convection scheme used in the ECMWF model is a so-called mass flux scheme described by Tiedtke [1989], Gregory et al. [2000], and Bechtold et al. [2004]. It bases its estimate of convective mass fluxes and associated heating, moisture and momentum transport on input column profiles of temperature and humidity. A theoretical “parcel” of air consisting of the mixed-layer mean temperature and humidity properties is lifted vertically to test for conditional instability in the column. The test parcel for deep convection is given small temperature and humidity excesses of 0.2 K and 0.1 g kg⁻¹, respectively, in addition to a positive upward velocity of 1 m s⁻¹. These perturbations, which are temporally and spatially invariant, crudely account for some estimate of subgrid-scale variability, and implicitly assume that temperature, humidity and velocity perturbations are always positively correlated in the test parcel.

If positive CAPE is found, and any CIN present is insufficient to decelerate the parcel to a zero vertical velocity within 200hPa of the parcel initiation level, essentially constituting the scheme’s convective “trigger”, then the scheme will produce deep convective heating rates proportional to the CAPE. During the ascent the parcel entrains mass from the environment at a rate E according to

\[ E = \frac{M}{\rho} \frac{1}{q} \left( \nabla q + \omega \frac{\delta q}{\delta p} \right) \]

where \( M \) is the updraught mass-flux, \( \epsilon \) a fixed turbulent entrainment rate of \( 1.2 \times 10^{-4} \) m⁻¹, \( q \) is the grid-mean humidity mixing ratio, and \( \omega \) the vertical velocity. The first term represents a fixed entrainment into updraughts while the second term related to moisture convergence only operates in the lower troposphere where this term is positive. The grid-mean humidity is mixed uniformly into the updraught. Note that the pre-existing cloud condensate in the grid column is ignored by the scheme, affecting neither the calculation of parcel buoyancy, nor the total water entrained into updraughts, and also that this description of entrainment is not valid after ECMWF forecast model cycle 32r3 was implemented on 7 November 2007.

Increasing the boundary layer humidity (and/or temperature), and thereby the parcel original equivalent potential temperature and associated CAPE, will make deep convection more likely to occur and stronger when it does. Likewise the entrainment of dry or cold air reduces the buoyancy of convective plumes and is likely to suppress convection [e.g., Parsons et al., 2000; Derbysire et al., 2004]. Changes in CAPE and CIN resulting from boundary layer and free tropospheric thermodynamical perturbations are considered by Parker [2002], who pointed out that the CAPE behaves fairly linearly in response to such perturbations, but the behavior of CIN (and hence convective triggering) is more subtle, depending critically on the depth
the CIN layer in addition to the magnitude of the negative buoyancy perturbation.

[12] Of direct relevance to this work is the study of Lopez and Moreau [2005], who analyze the Jacobians of the operational convective scheme, that is, the sensitivity of the scheme’s output tendencies to perturbations in input humidity and temperature (their Figures 10 and 11). They demonstrate that humidity perturbations have by far the greatest impact in the boundary layer, through their influence on the test parcel’s properties. There is also a secondary peak of sensitivities in response to humidity perturbations in the lower troposphere, where entrainment affects shallow and deep convection significantly. This agrees broadly with the cloud resolving model investigations of Tompkins [2001], which also show the greatest sensitivity to boundary layer humidity.

[13] Lopez and Moreau [2005] also show that boundary layer humidity perturbations have greater effect than temperature. This is expected since humidity fluctuations in the boundary explain most of the variability in equivalent potential energy and above the boundary layer virtual temperature fluctuations are dissipated efficiently by gravity waves. Therefore this work neglects temperature fluctuations for simplicity and only considers humidity fluctuations.

### 2.2. Stochastic Convection Parameterization Error Scheme

[14] The impact of the new scheme is assessed by comparison to the current operational stochastic physics scheme described by Buizza et al. [1999] that accounts for parameterization error by perturbing output tendencies. The scheme of Buizza et al. [1999] is modified to only apply the stochastic perturbations to the convection scheme, although sensitivity tests show that the impact of this modification is very minor since the convection scheme perturbations dominate in the tropics. At each model time step the temperature, humidity, and velocity tendencies are multiplied by a uniformly distributed random number lying between 0.5 and 1.5. Thus parameterization error can be as large as 50% of the scheme’s total tendencies.

[15] The scheme of Buizza et al. [1999] does not apply different random perturbations for each grid box and model time step, but instead introduces spatial and temporal coherence by applying the same random perturbation over a 10 by 10 degree grid and for a six hour period. The justification given for the use of this coarse grid of 648 tiles and the longer stochastic physics time step is that unaccounted organized systems that are represented by the scheme have intrinsic spatial and temporal scales that may span more than one model time step and more than one model grid point.

[16] For each six-hour stochastic physics time step, the average perturbation applied by the scheme over the 648 tiles is likely to lie very close to zero, relative to the standard deviation of the perturbation magnitude, but is not identically zero, which is only the case averaged over an infinite number of time steps. For simplicity in the text, the scheme is referred to as applying a zero-mean tendency perturbation, which is a very good approximation.

[17] The experiment using this modified scheme of Buizza et al. [1999] will be referred to as OUT-PARAM, since the stochastic convection scheme perturbs the output tendencies to account for PARAMeterization error.

### 2.3. Stochastic Convection Subgrid Perturbation Scheme

[18] The new stochastic scheme to represent convective uncertainty because of subgrid variability in humidity is now described. If the subgrid variability of humidity and temperature is to be taken into account by the convection, the nature of that variability needs to be specified. In fact many GCM schemes already contain implicit assumptions concerning subgrid variability, such as the cloud water adjustment made in radiation schemes [e.g., Cahalan et al., 1994], or estimates used in the evaporation of rainfall [e.g., Jakob and Klein, 2000].

[19] Alternatively explicit subgrid information can be provided by parameterization schemes which adopt a low-order probability density function (PDF) to describe the fluctuations of temperature and/or total water (humidity + cloud water). These schemes can be diagnostic, such that the PDF descriptors are defined in terms of other prognostic variables (the diagnostic cloud schemes of Smith [1990] and Bony and Emanuel [2001] are examples), or prognostic, such that the PDF descriptors themselves evolve in time [e.g., Golaz et al., 2002; Tompkins, 2002].

[20] While the cloud scheme in the ECMWF integrated forecast system model does not explicitly employ a statistical scheme framework, the central parameterization processes assume a uniform distribution for the humidity distribution in the clear sky portion of the grid cell [Jakob, 2000]. The distribution was employed previously in a cloud scheme by LeToulu and Li [1991] and has the advantage of simplicity, but the review of observations by Tompkins [2002] makes it clear that an uni- or multi-modal distribution with tails, that can also express positive or negative skewness, would be a more accurate description of nature. This information will be used to provide consistent perturbations of humidity variability for the convection scheme. In the cloudy region of the grid cell the cloud scheme disregards variability (the total water PDF is a delta function), but this is in any case irrelevant since the convection scheme only considers subsaturated humidity profiles.

[21] The clear sky distribution is simply determined. If the grid mean relative humidity exceeds RH_{crit}, the fixed threshold at which the parameterization starts to form cloud, then it is assumed that the variability of the clear sky PDF is such that the moistest part (the clear-sky distribution maximum q_{max}), is equal to the saturation mixing ratio q_{sat}. Although the model’s cloud physics permits clear-sky supersaturated states for temperatures below 235K [Tompkins et al., 2007], this is not accounted for here, as the model’s convection scheme clips input humidity profiles to the saturation value.

[22] From the assumption of a uniform PDF, the distribution minimum, the driest value of mixing ratio found in the grid cell, is given by q_{min} = q_e - (q_{sat} - q_e), where q_e is the mean humidity in the clear-sky portion of the grid-cell (the cloud “environment”). Following Tiedike [1993], RH_{crit} is set to 0.8 throughout most of the troposphere, but increases toward unity when the pressure p exceeds 80% of the surface value p_{surf} according to

$$RH_{crit} = 0.8 + H\left(p - 0.8p_{surf}\right)5\left(p/p_{surf} - 0.8\right)^2,$$  \hspace{1cm} (2)

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where $H$ is the Heaviside function. In the cloud scheme $RH_{\text{crit}}$ also increases near to the tropopause, but this is irrelevant here since the convection scheme does not allow entrainment at these levels.

[23] In the case of clear sky conditions, when the mean relative humidity is less than the critical threshold $RH_{\text{crit}}$ for cloud formation, $q_{\text{max}}$ cannot equal the saturation value by definition. A diagnostic distribution width must be defined, since it cannot be determined from the available prognostic variables. The simplest assumption is that the distribution width is constant in time and space for all relative humidity values below $RH_{\text{crit}}$. The distribution minimum and maximum values are thus given by $q_{\text{min}} = q_e - (q_{\text{sat}} - q_{\text{crit}})$, where $q_{\text{crit}} = RH_{\text{crit}} q_{\text{sat}}$ and likewise $q_{\text{max}} = q_e + (q_{\text{sat}} - q_{\text{crit}})$. This lack of memory of the PDF evolution in clear sky conditions is the disadvantage of a cloud scheme where the prognostic quantities are integrated statistics such as cloud cover or cloud water mass mixing ratio (see Tompkins [2002] for full discussion).

[24] Combining the above two cases, the minimum and maximum mixing ratios in the clear sky portion of the grid box are:

$$q_{\text{min}} = q_e - (q_{\text{sat}} - \text{MAX}(q_e, q_{\text{crit}})).$$  

$$q_{\text{max}} = \text{MIN}(q_{\text{sat}}, q_e + (q_{\text{sat}} - q_{\text{crit}})).$$  

[25] In summary, the distribution in the clear sky component is fixed for $RH < RH_{\text{crit}}$ and then progressively reduces with increasing $RH$ once $RH$ exceeds $RH_{\text{crit}}$ consistently with the condensation process occurring. Note that at very dry values of $RH$ the distribution width is reduced to ensure that $q_{\text{min}}$ does not become negative.

[26] The mean environmental humidity $q_e$ in the above equations is derived in terms of the grid-box mean humidity $\overline{q}$ using the relationship $\overline{q} = C q_{\text{sat}} + (1 - C) q_e$, where $C$ is the cloud fraction, and which assumes no supersaturation exists within the cloudy portion of the grid-cell.

[27] The humidity distribution is sampled by taking a single random number $r \in [0, 1]$ for each grid column which is uniformly distributed to be compatible with the assumed underlying distribution for humidity. In this way, no further separate assumptions concerning subgrid-scale variability are introduced. The aim should be to introduce one central key PDF for subgrid variability, and to use this consistently in all model parameterization components, such as convection, clouds, radiation, and in this case stochastic physics. It is noted, however, that other distribution shapes with tails, such as the lognormal or Gaussian are likely to be more appropriate, both for the stochastic physics and also for describing the source and sink terms in the cloud microphysics.

[28] If the random number falls into the clear sky part of the domain ($0 < r < 1 - C$) then the input humidity $q_{\text{in}}$ is set to

$$q_{\text{in}} = \frac{r}{1 - C} (q_{\text{max}} - q_{\text{min}}) + q_{\text{min}}.$$  

[29] Since the convection scheme does not take cloud water into account, the input humidity is set to $q_{\text{sat}}$ if $r > 1 - C$. The latter possibility of the convective parcel sampling the cloudy region in leads to a positively skewed rather than a Gaussian input perturbation PDF, as illustrated in the example of Figure 1. Although clouds start to form at relative humidities of 80% in the mid-troposphere, the fact that they are prognostic entities implies that they can and do continue to exist in grid boxes that are substantially drier than this. Sampling the relatively rare cloudy regions in dry grid boxes at a pressure level of 700 hPa (Figure 1a) leads to the minor distribution tail of positive anomalies. The boundary layer in the tropics is generally moist and, often cloud free, thus the distribution is symmetrical at 950 hPa (Figure 1b).

[30] For each grid box the same random number is used throughout the atmospheric column. If, for instance, the boundary layer parcel is taken from the driest part of the distribution, that parcel will also rise through the driest portion of each subsequent grid cell during its ascent. This assumption is essentially one of maximum overlap of subgrid-scale humidity fluctuations, but there is no reason why future work could not also investigate other overlap assumptions [e.g., Pincus et al., 2005]. To ensure the setup for the new stochastic convection scheme is equivalent to that of the OUT-PARAM experiment, the random perturbations to the humidity input profiles are also applied on the same 10 by 10 degree grid and six hour stochastic physics
time step. Note that, similarly to the modified scheme of Buizza et al. [1999], it is a good approximation to state that the mean humidity input perturbation is zero averaged over the stochastic physics’ tiles each 6 hour period.

[31] It is recalled that the standard convection scheme does not initialize the test convective parcel with grid-mean values but actually adds an offset. In the new scheme, the humidity offset of 0.1 g kg$^{-1}$ is still applied after the humidity distribution is sampled. If this were not the case, the drier boundary layer parcels in the revised scheme would result in reduced effective CAPE, and the activity of the deep convective would be suppressed at the expense of increased grid-scale convection, possibility affecting forecast skill [Tompkins and Jung, 2003]. Temperature and humidity fluctuations are thus on average positively correlated in the new stochastic physics scheme, but this correlation is extremely weak, since the humidity distribution width in the boundary layer, and thus the standard deviation of the humidity perturbation, greatly exceeds the magnitude of the constant offset of 0.1 g kg$^{-1}$ (by more than an order of magnitude in the tropics for example, see Figure 1a). Future developments of statistical cloud schemes could instead provide a joint probability density function of temperature, humidity and velocity, such as those of Golaz et al. (2002), permitting more realistic cross-correlations.

[32] The experiments using this new scheme will be collectively referred to as IN-SUBGRID, since the stochastic convection scheme perturbs the INput profiles to account for SUBGRID-scale fluctuations.

2.4. Experimentation

[33] An array of ensemble prediction system (EPS) experiments was conducted consisting of 10 day forecast integrations using model cycle 31R1 (the version used operational in the system 3 version of the ECMWF seasonal forecasting system Anderson et al. [2007]), for 23 cases spread 16 days apart starting on the 1 May 2004. The forecast ensemble consists of 30 members at T255L40 resolution, with a further control integration conducted at the same resolution but with the perturbations disabled.

[34] The array of experiments are summarized in Table 1. In addition to OUT-PARAM integration, three experiments are conducted using the new scheme, IN-SUBGRID. In the first, the subgrid-scale humidity distribution is allowed to affect the convection only below 800 hPa in the planetary boundary layer, and is referred to as IN-SUBGRID-BOUNDLAY. The main effect is on the convective parcels’ initial properties, and entrainment will play a very minor role. The second experiment in contrast perturbs the profiles above 800 hPa in the free troposphere (IN-SUBGRID-FREETROP). Thus the properties of the test parcels raised from the boundary layer are unaffected, and only the entrainment mechanism operates.

[35] It is likely that the IN-SUBGRID-BOUNDLAY and IN-SUBGRID-FREETROP experiments will result in sharp humidity gradients at 800 hPa in the profiles passed to the convection scheme. As humidity at this level only affects convection through the entrainment process this will not result in any unphysical behavior of the scheme, and the gradients do not necessarily imply that the profiles are unrealistic, since actual atmosphere profiles often exhibit moist or dry layers with sharp gradients at their boundaries [e.g., Mapes and Zuidema, 1996]. The third experiment combines the former two by applying perturbations to the full profile throughout the atmospheric column, and thus both parcel original and subsequent entrainment properties are affected. This experiment is referred to as IN-SUBGRID-FULLPROF.

[36] Since the new scheme was intended to supplement the scheme of Buizza et al. [1999] for parameterization error, a final experiment combines the two schemes, applying both input perturbations throughout the atmospheric column in addition to output perturbations, and is referred to OUT-PARAM-IN-SUBGRID. Unless stated explicitly, no perturbations to the initial conditions are made, and the ensemble spread results entirely from the respective stochastic convection schemes.

3. Results

3.1. Ensemble Spread

[37] Figure 2a gives the average 700-hPa RMS temperature error of the control forecast (the forecast to which stochastic perturbations are not applied) compared to the ECMWF operational analyses at the day two range averaged over the same 23 dates for which the EPS experiments are performed. The RMS Errors are mostly in the 0.5 to 2 K range throughout much of the extra-tropics, while smaller errors are seen in the tropics.

[38] Ideally, ensemble spread would be of a comparable magnitude to this error at the same forecast range. In the operational EPS system, much of the ensemble spread in midlatitudes results from the perturbations applied to the initial conditions, defined using singular vectors [Palmer et al., 1998]. To illustrate this Figure 2b shows the day 2 spread interensemble spread that results from an EPS experiment using only initial perturbations and with the stochastic physics schemes disabled. The spread is quite uniform in the extra-tropics, of a comparable magnitude to the RMS error, but generally larger, in the 1 to 2 K range, and in many regions exceeds the forecast errors. At later forecast ranges, this situation reverses, with the spread generally smaller than RMS errors (not shown), a behavior which is well known [Buizza et al., 2005], and is one of the original motivations for the development of the stochastic physics scheme, to add spread at later forecast ranges. Moreover, in the tropics the singular vectors are designed.
to target cyclones only, and the spread in this region using initial condition perturbations is significantly underestimated, also demonstrating a requirement for a stochastic treatment of the convective process.

Comparing the stochastic convection experiments (Figure 3), the spread in temperature at day 2 is generally far smaller in the extra-tropics than that resulting from the application of initial perturbations. For example, in the OUT-PARAM case the spread is limited to less than 0.5 K and at this forecast range the impact is quite uniform throughout the tropics and extra-tropics.

Considering the new schemes which consider sub-grid uncertainty, perturbing the boundary layer (IN-SUBGRID-BOUNDLAY, Figure 3b) has a greater effect on ensemble spread than applying humidity perturbations to the convection scheme in the mid-troposphere (IN-SUBGRID-FREETROP, Figure 3c), and relatively, the spread is larger over the oceanic regions. Perturbing the tropospheric humidity has the greatest effect over the deep convective regions of central America and Africa.

Perturbing the input throughout the column (IN-SUBGRID-FULLPROF) leads to increased spread as expected (Figure 3d), but the effect is not linear, with the spread in IN-SUBGRID-FULLPROF generally around 20 to 30% less than the sum of the separate IN-SUBGRID-BOUNDLAY and IN-SUBGRID-FREETROP cases. At this day two range, the spread from the IN-SUBGRID-FULLPROF experiment is similar to modified Buizza scheme OUT-PARAM in the tropics and sub-tropics, but is less effective in midlatitudes. Finally, the greatest spread is achieved

Figure 2. (a) Day 2 RMS error (compared to the daily analysis) in 700-hPa temperature averaged over 23 cases from the ensemble prediction system control (unperturbed) forecast. (b) Day 2 interensemble “spread” (RMS differences) in 700-hPa temperature averaged over 23 cases resulting from the application of initial perturbations and with all stochastic physics schemes disabled.

Figure 3. Day 2 interensemble 700-hPa temperature spread for (a) OUT-PARAM, (b) IN-SUBGRID-BOUNDLAY, (c) IN-SUBGRID-FREETROP, (d) IN-SUBGRID-FULLPROF, and (e) OUT-PARAM-IN-SUBGRID for the same 23 cases.
when both parameterization error and subgrid-scale variability are accounted for (OUT-PARAM-IN-SUBGRID, Figure 3e), and throughout most of the tropics the magnitude is that required although over the tropical continents the spread appears to be overestimated (refer to Figure 2a).

The drawback of examining spread in temperature, is that inter-member differences are communicated rapidly over wide areas by gravity wave propagation. In contrast, the spread in humidity at 700hPa reveals exactly where the stochastic convection schemes have the most influence (Figure 4), which is seen to coincide with the convective regions very closely, with the tropical continents, West Pacific, and the Intertropical and Southern convergence zones clearly demarked. Similar to the spread in temperature, the assessment of parameterization error in OUT-PARAM leads to more interensemble spread than the uncertainty because of the subgrid-scale fluctuations (IN-SUBGRID-FULLPROF), but both are of a similar magnitude.

The increased spread for forecast ranges of 5 and 10 days (Figure 5) shows that the mainly tropical influence of the scheme in the day 0–3 range spreads to midlatitudes by the medium range. The magnitude of the spread in midlatitudes is still substantially less in these extended ranges than that rendered by singular vector-derived initial condition perturbations (not shown).

### 3.2. Probabilistic Skill Scores

For reasons of brevity, the probabilistic skill of the EPS experiments is assessed in terms of the continuous ranked probability skill score [Hersbach, 2000; Candille and Talagrand, 2005], which is identical to the integral over Brier scores for all thresholds and for a deterministic forecast is identical to the mean absolute error. The skill scores are shown for temperature in the extra-tropics at 850 hPa, and the zonal and meridional components of the winds at 200 hPa and 850 hPa in the tropics (Figure 6).

The lower impact on temperature spread is reflected in a lower probabilistic skill for the IN-SUBGRID schemes relative to the parameterization error scheme OUT-PARAM. There is little difference in skill between the IN-SUBGRID cases, since the midlatitude contribution to spread is limited. However in both the Northern (Figure 6a) and Southern (Figure 6b) hemispheres, taking the additional uncertainty of subgrid-scale variability in account improves the skill of assessing only parameterization uncertainty (OUT-PARAM-IN-SUBGRID).

For the wind scores in the tropics, the greatest impact from the IN-SUBGRID scheme derives from the humidity perturbations made in the boundary layer (Figure 6c–6f). This was expected from the fact that the convection scheme is most sensitive to boundary layer perturbations. The probabilistic skill for both the meridional and zonal components is improved in the short range, when both schemes are combined, but this improvement is only sustained in the medium range and beyond for the meridional winds. For both the 850-hPa and 200-hPa zonal winds, the probabilistic scores are worse for the combined OUT-PARAM-IN-SUBGRID scheme after day 3 than the OUT-PARAM scheme in isolation.

To discover the origin of the skill deterioration of the zonal winds in the combined experiment, Figure 7 shows time series of the spread of the ensemble around the ensemble mean and the RMS error of the ensemble mean for the 200-hPa winds (the charts for the 850-hPa levels are similar and thus not shown). The lower lines showing ensemble spread confirm the earlier analysis of temperature and humidity, with the IN-SUBGRID schemes generating

![Figure 4](image-url)  
**Figure 4.** Day 2 interensemble 700-hPa humidity spread for (a) OUT-PARAM and (b) IN-SUBGRID-FULLPROF for the same 23 cases.

![Figure 5](image-url)  
**Figure 5.** Same as Figure 3 but for IN-SUBGRID-FULLPROF at forecast ranges of (a) day 5 and (b) day 10.
less 200-hPa zonal wind spread than OUT-PARAM in the tropics at day 2. The growth of the spread during the forecast is also lower.

The lower growth in spread could result from smaller tendency perturbations in the IN-SUBGRID schemes, and thus Figure 8 compares the first time step RMS zonal wind tendency perturbation averaged across the tropics for OUT-PARAM and IN-SUBGRID-FULLPROF. The profiles are similar, with a local peak in the upper troposphere coinciding with the deep convective detrainment level, and larger values in the boundary layer. Differences in the profiles, such as the shift in the detrainment peak, result from the diverse approaches. For example, as the OUT-PARAM scheme perturbs the convective output tendencies, it scales

**Figure 6.** Continuous ranked probabilistic skill scores for the 850-hPa temperature in the (a) Northern and (b) Southern hemispheres (excluding the tropics) and the (c, e) zonal and (d, f) meridional components of the (c, d) 850-hPa and (e, f) 200-hPa tropical winds for the EPS experiments.
the output profile and is unable to change the convective cloud top height, while the input perturbation used in the IN-SUBGRID methodology may alter the distribution of cloud top heights and associated detrainment profiles.

It is recalled that the parameterization error assessment method OUT-PARAM generates large perturbations in convective tendencies of up to 50%. The stochastic convective schemes that take input variability into account (IN-SUBGRID) have the potential to provide even larger perturbations since they can switch the entire convective scheme on or off for a given location. However, averaged across the tropics the RMS zonal wind tendency perturbation of IN-SUBGRID-FULLPROF are in fact roughly half the magnitude of OUT-PARAM throughout most of the troposphere (Figure 8). One of the reasons is that the uncertainty due to subgrid humidity variability is greatest in atmospheres drier than 80% RH; recall that as the RH approaches saturation, the diagnosed width of the subgrid humidity distribution, and thus also the input perturbation magnitude, reduces to zero. Since convection is most likely to occur in moist or saturated atmospheres [e.g., Raymond, 2000] the implication is that the input perturbation will be smallest when convective heating rates are largest, which is seen to be the case in Figure 9. This is a contrasting relationship to the schemes of Buizza et al. [1999] and Lin and Neelin [2003] for assessing parameterization error, which provide the largest perturbations when convective heating rates are greatest, implying a greater potential for interensemble divergence.

The lower probabilistic skill for the zonal winds in the latter midrange forecast ranges for OUT-PARAM-IN-SUBGRID compared to OUT-PARAM derives from the larger RMS error of the ensemble mean at these ranges (Figure 7), which is not compensated for by the modest increase of spread in the combined system. The 700-hPa temperature continuous ranked probability skill scores, RMS error, and spread have the same relative relationships between the EPS experiments as the winds in the tropics (not shown).

### 3.3. Nonlinear Issues

The question that remains to be answered is why the tropical zonal wind (and temperature) ensemble mean RMS errors in the combined OUT-PARAM-IN-SUBGRID experiment are greater than the experiment which take only the parameterization errors into account (OUT-PARAM). A clue is found when the ensemble spreads around the respective ensemble means (Figure 7) are compared to the spreads calculated around the control forecasts (Figure 10). The control is defined as an unperturbed forecast, and since the experiments are run for the same dates, it is identical for all experiments.

For the early forecast times the RMS error of the control and ensemble mean are very similar. For later forecast ranges taking the ensemble mean acts as a nonlinear filter for the unpredictable scales and thus the ensemble mean has a smaller error than the control. Correspondingly,

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**Figure 7.** RMS of ensemble mean (upper lines marked “M”) and spread around the ensemble mean (lower lines marked “S”) of the 200-hPa zonal wind (m s\(^{-1}\)) for the EPS experiments as a function of forecast day.

**Figure 8.** First time step RMS zonal wind tendency perturbation averaged across the tropics (latitudes less than 20°) for a single forecast using the OUT-PARAM (thin solid line) and IN-SUBGRID-FULLPROF (thick dotted line) schemes.

**Figure 9.** Normalized RMS humidity perturbation applied at 700 hPa at day 1 for IN-SUBGRID-FULLPROF as a function of local precipitation rainfall generated by the convective parameterization (mm day\(^{-1}\)).
we expect the spread around the ensemble mean to be smaller than around the control forecast and this is indeed the case when comparing the spreads in Figures 8 and 10 for each experiment. However, the relative increase of spread around the control compared to that around the ensemble mean is much more pronounced for OUT-PARAM-IN-SUBGRID than for OUT-PARAM. This implies that the stochastic perturbations applied in IN-SUBGRID-OUT-PARAM are producing a marked shift (bias) in the ensemble relative to the unperturbed control. This bias seems to be detrimental to the forecast as the increased RMS error of the ensemble mean indicates. The potential to introduce not only an increase in variability but also to alter the model bias is a potential strength of stochastic parameterizations [e.g., Palmer, 2001], but it is obvious that this will only be beneficial if this acts to reduce rather than increase the relevant model error.

Why should the IN-SUBGRID approach of applying stochastic perturbations to the convective scheme input produce such a bias in the ensemble behavior, when compared to the OUT-PARAM scheme, that directly perturbs the output? The answer lies in the inherent nonlinearity of the convective scheme. The scheme of Buizza et al. [1999] for parameterization uncertainty ensures that the mean of the parameterization tendency perturbations is zero. Similarly, the new scheme that assesses uncertainties because of subgrid-scale variability was also designed such that the mean of the input perturbation is zero. However, unlike the approach of Buizza et al. [1999], this does not automatically imply that the perturbations to the parameterization output tendencies are zero on average, since the convection scheme is a highly nonlinear operator. Thus zero-mean input perturbations can lead to finite mean output perturbations. For example, for a given atmospheric thermodynamic profile a positive boundary layer humidity perturbation may lead to a small enhancement of convective mass-fluxes, while the same magnitude negative perturbation may prohibit deep convection altogether. Examining the PDF of convective rain-rate in the control and a perturbation forecast of IN-SUBGRID-FULLPROF shows that there is a shift in the PDF, that exceeds the interensemble variance, with increased incidence of smaller rain-rates (Figure 11).

Introducing a shift to a scheme output does not necessarily affect model probabilistic skill detrimentally, as theoretically it could also offset an existing bias and equally improve the model. However, since model parameterization schemes are tuned and developed in an attempt to maximize forecast skill, a non-zero-mean perturbation to their tendencies is more likely to deteriorate the skill of a particular forecast, as was the case for the post day-3 range tropical zonal wind scores for the OUT-PARAM-IN-SUBGRID experiment.

Interestingly, the deterioration in skill for the IN-SUBGRID schemes is not evident when deterministic skill is assessed. A measure of deterministic skill is the RMS error of the individual forecasts. While the unperturbed forecast has the smallest error, the error of IN-SUBGRID-FULLPROF is only slightly larger and much less pronounced than that of OUT-PARAM (Figure 12). Thus the larger perturbations in OUT-PARAM dominate over the increased bias of IN-SUBGRID-FULLPROF.

As the treatment of physical processes steadily improves it is possible that the nonlinearity could become more important when designing stochastic schemes that perturb input profiles. Moreover, the impact of nonlinearity was quite probably minimized by the scheme outlined here that was relatively ineffective at producing spread in near-saturated profiles (refer to Figure 9). Moreover, the version of the convective scheme used here is relatively insensitive to mid-tropospheric humidity [Derbyshire et al., 2004], a characteristic that has been substantially improved with the implementation of a revised scheme in model cycle CY32R3, which became operational on 7 November 2007 (P. Bechtold, personal communication). Thus a future stochastic convective scheme that permits a perturbed input may find it necessary to either adjust the mean, variance and high-order moments of the input PDF such that the perturbed output has a zero mean, or to apply a post-scheme...
adjustment to reset the mean of the tendency perturbations to zero.

4. Conclusions

[57] This work has outlined a stochastic convection scheme, whereby the input profile of humidity provided to the convection scheme is not simply the grid-mean value, but instead is randomly sampled from an underlying distribution that accounts for subgrid-scale variability. Thus the forecast uncertainty because of humidity fluctuations on scales below the truncation scale of the model is accounted for.

[58] The potential advantage of such a scheme is that subgrid thermodynamic variability, and thus also the input perturbations, can be directly linked to the local dynamical and thermodynamical conditions in a physically sound way, for example by using a prognostic statistical cloud scheme. Moreover, perturbing the parameterization input ensures the output perturbations are thermodynamically and dynamical self-consistent.

[59] Since the ECMWF forecast model does not currently have a prognostic statistical cloud scheme, the stochastic convection scheme outlined here determined the subgrid humidity fluctuations using the prognostic variables of humidity, cloud water and cloud cover provided by the scheme of Tiedtke [1993]. Dynamical processes, such as turbulence, that affect these latter prognostic quantities will also affect the humidity fluctuations indirectly. A uniform PDF shape for humidity fluctuations in the clear sky was adopted to be consistent with the assumptions used in the cloud scheme.

[60] The stochastic scheme was introduced into the ECMWF EPS system, and preliminary tests showed that the humidity variability has more importance in the boundary layer, determining the convective test parcel’s original properties, than above the boundary layer where it affects the convective mass fluxes by its influence on entrainment.

[61] The new scheme was compared to the operational stochastic physics scheme of Buizza et al. [1999], modified to only operate on convection, and which perturbs parameterization output to account for parameterization error. It was found that accounting for uncertainty because of subgrid-scale humidity variability had a smaller impact on interensemble spread than the assessment of parameterization error.

[62] Adding the new scheme to the operational stochastic physics scheme, to account for both subgrid uncertainty in addition to parameterization error, generally improved the EPS skill scores in the short and medium range in midlatitudes. However, in the tropics the results were mixed, with the probabilistic skill scores for zonal winds and temperatures being affected detrimentally by the new scheme in the medium (post day-3) range.

[63] The reduction in skill for some parameters in the tropics was shown to be due to an increase in bias of the ensemble mean introduced by the new scheme. It was pointed out that the operational stochastic physics scheme of Buizza et al. [1999] introduces zero-mean perturbations to the convective scheme tendencies by construction. The same is not true for the new scheme outlined here, since the convection scheme is a highly nonlinear operator, so that a zero-mean input perturbation PDF does not lead to a zero-mean output. For any parameter for which the resulting shift increases ensemble-mean bias, deteriorating probabilistic skill is likely to ensue.

[64] This could potentially restrict the development of such approaches in future as parameterization schemes are steadily developed and improved to reduce biases and maximize forecast skill. If this methodology of accounting for subgrid-scale uncertainty is therefore to be ultimately successful, an approach will be required to either determine the nature of the input perturbation PDF that leads to a non-biased output perturbation PDF for highly nonlinear schemes, or empirical post-scheme adjustments will be necessary to impose this condition.

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